The 'starting plume' in neutral surroundings

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The advancing front of a buoyant plume which is being established in uniform surroundings has some properties in common with the plume, while in other respects it behaves more like a 'thermal' released from rest. The solutions for these two cases cannot be matched directly, since the dependence of velocity on height is different. It is shown here that a similarity solution, which is consistent with the equations describing both parts of the flow, can be obtained once it is recognized that the velocity of the front may be less than that of the steady plume. The cap moves with a constant fraction of the plume velocity at the same level, and the total buoyancy in the cap is increasing, so a modification of the simple relations for thermals is required.

This prediction is verified experimentally, and numerical values for the ratio of the velocities and the rate of increase of the cap radius with height determined. The extreme front of plume cap advances at about 0·6 times the mean velocity on the axis of a steady plume, and it spreads at just over half the angle of a thermal. This implies a smaller rate of entrainment and therefore a smaller rate of dilution per unit height compared with a thermal, especially since about half of the fluid mixed into the cap comes from the plume below. The model allows one to estimate the time necessary for convection from a known steady source on the ground to lead to the formation of a cloud.

1. Introduction

In the study of convection in the atmosphere, and the mixing processes in and below cumulus clouds, two main types of theoretical models have been proposed. The first of these is the plume or jet model, in which the cloud is supposed to behave like a steady turbulent plume which entrains the environmental air round its sides. The second group consists of the bubble theories, which trace the development in time of elements of buoyant fluid; these can be thought of either as comprising the whole of the cloud, or as smaller elements within it ('thermals') comparable in size with individual cumulus towers.

Each of these models has certain obvious disadvantages. The plume theories disregard any development in time of the depth or width, and they neglect any possible mixing at the top of the cloud. There is reason to believe that they can thereby seriously underestimate the mixing (see, for example, the recent survey by Squires 1962). The bubble theories, on the other hand, ignore the possibility of a relatively steady, continuing source of heat and moisture below cloudbase; a continuous renewal of cloud by condensation seems to be implied by the existence of the sharp flat bases which are observed. In fact it would seem that an improved model of cloud growth should include features from both the earlier types of theory: the supply of buoyancy from below, entrainment at the sides and mixing at the top must all be considered.

The necessity for such a model becomes clearer on those occasions when the plume of hot air below the cloud is made visible because it contains smoke. Figure 1, plate 1, is a photograph of a cloud which has formed on top of a smoke plume produced by burning grass (similar observations have been reported over fires in stubble or sugar cane). It is suggested in this paper that even in cases where the warm column is not visible below the condensation level, the continuing supply of buoyancy can have a significant effect on the motion in small cumulus clouds.

The production of extra buoyancy by the release of latent heat will of course also have to be taken into account in a full solution of the problem, though this is outside the scope of the present discussion. Several papers have appeared which show the limitations of the plume model in this case; for example Morton (1957) has suggested a mechanism whereby large condensing clouds can become independent of the plume from which they formed. Recently, Mason & Emig (1961) have made a theoretical study of cloudy convection in which they use some ideas from plume theory (uniform properties in the horizontal plane) but formulate the equations of motion in terms of the parcel model. The physical basis of the implied assumptions is not stated specifically, but their model is more complicated and less well defined than the one which will be discussed here.

As a first step towards the understanding of this complex motion, we set ourselves here a simpler problem. This is to investigate theoretically and experimentally the motion near the advancing front of a vertical plume, which is being established in neutral surroundings by suddenly emitting buoyant fluid at a constant rate from a point source. This model will therefore be strictly applicable to dry convection, before a cloud has formed, but it will probably also be relevant for small clouds in which condensation effects never become dominant. The flow obtained when such a plume has been in existence for a short time is pictured in figure 2, plate 2; this shows two photographs taken 1 sec apart of a plume of dyed salt solution moving downwards through fresh water. Note that the shape of the starting plume remains the same while the size is increasing.

It will be shown that a similarity solution may be used to describe this motion. In order to link the derivation more closely to previous models it will be assumed that the flow some distance behind the front is the same as that existing in a steady plume, while the motion in the cap or front itself is more like that in a 'thermal' released from rest. The qualitative reason for this idea is obvious from figure 3, plate 3, in which the supply of dye in a starting plume has been momentarily interrupted to show the nearly spherical 'front' and the steady plume behind. At first sight, any similarity theory based on this hypothesis would seem to be out of the question, since the functional dependence of velocity on distance is different for a plume and a thermal of constant total buoyancy. It must be recognized, however, that the velocity of the cap need not be the same as that of the plume fluid immediately behind it, so that there can be an increase of total buoyancy with time near the advancing front. This will change the character of the motion, and makes the two parts of the solution reconcilable, as shown below.

2. The steady plume

The well-known results for a steady plume in neutral surroundings will first be summarized. The approach of Morton, Taylor & Turner (1956), which is equivalent in neutral surroundings to the dimensional argument first used by Schmidt (1941), will be followed. Though the solution of Priestley & Ball (1955) may be more realistic near the origin (since it assumes a finite velocity there instead of the infinite value arising from the similarity solution), the various forms approach one another at greater heights, and it seems preferable to use the simplest here.

Let us make the common assumption that the profiles of mean velocity and density differences are Gaussian, with the same length scale b and maximum values $u(z)$ and $\Delta(z)$, which are functions of the height z above a virtual origin. Then

$$\left.\begin{aligned} u(z,r) &= u(z)\, e^{-r^2/b^2}, \\ g(\rho_0-\rho)/\rho_0 &= \Delta(z,r) = \Delta(z)\, e^{-r^2/b^2}, \end{aligned}\right\} \tag{1}$$

where ρ_0 and ρ are the densities outside and inside the plume. The entrainment principle used by Morton *et al.* is that the inflow velocity at any height is proportional to the velocity scale at that height, so the rate of entrainment of volume per unit height is $2\pi b\alpha u(z)$, α being defined as the 'entrainment constant'. The equations of conservation of volume, momentum and density deficiency integrated over the plume section may be written

$$\left.\begin{aligned} d(b^2u)/dz &= 2b\alpha u, \\ d(b^2u^2)/dz &= 2b^2\Delta, \\ d(b^2u\Delta)/dz &= 0. \end{aligned}\right\} \tag{2}$$

The third of these equations can be integrated immediately to give

$$Q = b^2u\Delta = (2/\pi)\times(\text{flux of }\Delta), \tag{3}$$

which is a constant. The solutions for b, u and Δ may be found in terms of Q and z in the forms

$$b = \tfrac{6}{5}\alpha z, \quad u = \tfrac{5}{6}\alpha^{-1}(\tfrac{9}{5}\alpha Q)^{\frac{1}{3}}\, z^{-\frac{1}{3}}, \quad \Delta = \tfrac{5}{6}Q\alpha^{-1}(\tfrac{9}{5}\alpha Q)^{-\frac{1}{3}}\, z^{-\frac{5}{3}}. \tag{4}$$

3. The vortex ring theory of thermals

In recent years, much detailed information has been obtained about the characteristics of thermals of constant total buoyancy in neutral surroundings. The laboratory experiments of Woodward (1959), for example, have shown that a thermal consists of a region of turbulent buoyant fluid whose radius is increasing linearly with distance as it mixes with its surroundings. A circulating motion is produced by the buoyancy, with a region of greater than mean velocity up the centre and a downward moving region at the edges. Woodward was able to plot out the flow pattern and show that the motion remains similar at all heights, and also to demonstrate that the addition of outside fluid takes place partly by mixing over the front of the thermal and partly by the drawing up of fluid from behind. In this sense it can be said that a thermal leaves no wake (of buoyant or turbulent fluid) behind it.

The motion described above is rather like that in a vortex ring, and in fact Turner (1957) has shown that the thermal may be regarded as a special case of a buoyant vortex ring. Levine (1959) too has based his theory of thermals on a spherical vortex, though he restricts the discussion to an element of fixed size, implying an equal turbulent transfer into and out of the element. We shall be concerned in this paper with a vortex which is growing by the turbulent entrainment of external fluid in an atmosphere at rest.

Provided the buoyancy remains constant, it follows from dimensional arguments that the dependence of velocity on the distance, say z_c, which the centre of the thermal has travelled is $v_c \propto z_c^{-1}$ both for a vortex ring and a thermal, and this is very different from the relation (4) for a plume. The spread in radius is linear with height, and the general vortex-ring theory has shown how the angle of spread can depend both on the buoyancy and the initial momentum given to the element, instead of being determined uniquely by the buoyancy as it is for a thermal. The more general theory must be used when it is desired to introduce assumptions which allow the buoyancy to change with time.

Several of the equations developed previously will hold unchanged in the present case. They will be repeated here for convenience of reference, but for details see Turner (1957). The momentum P of all the fluid moving with the ring or thermal is given by

$$P = \pi\rho' K R^2, \tag{5}$$

where K is the circulation, ρ' is the mean density and R is the 'mean radius' as defined by Lamb (1932). If it is further assumed that the distribution of vorticity in the cap remains similar at all heights, the velocity of the centre of the cap may be written as

$$dz_c/dt = v_c = cK/R, \tag{6}$$

where c is a constant, which is not necessarily the same as that for a thermal but should be not too different from it. It is convenient for the present purpose to eliminate K from (5) and (6) to give

$$P = (\pi\rho'/c)\, R^3 v_c. \tag{5a}$$

For a known total buoyancy $F = gV(\rho_0 - \rho')/\rho_0$ (where V is the volume of buoyant fluid, ρ_0 and ρ' are the densities outside and inside the thermal and g is the acceleration due to gravity), the time rate of change of P may be determined using the momentum equation.

$$dP/dt = \rho_0 F. \tag{7}$$

As usual we shall neglect all density differences in the inertia terms.

4. The front of a starting plume

A very small addition to the ideas described above leads to a possible picture of the front of a 'starting plume'. Suppose that the cap advances through stationary fluid and mixes with the surroundings there, but that the properties of the region behind it are given by the solution (4). Thus the fluid drawn up into the advancing vortex ring will now come from the *plume* instead of the environment, and will add extra buoyancy and momentum to it. The buoyancy distribution will therefore be different from that in neutral surroundings, since plume fluid will be fed through the centre directly to the front of the cap. It is also to be expected that the mean velocity pattern could be modified somewhat from that given by Woodward, but it will still have the same characteristics of a circulating motion with a high velocity up the centre.

It remains to put this idea into quantitative form. In the previous application, it was possible to assume that the buoyancy F was constant, and that P changed through the action of buoyancy alone (equation (7)). In the present case both

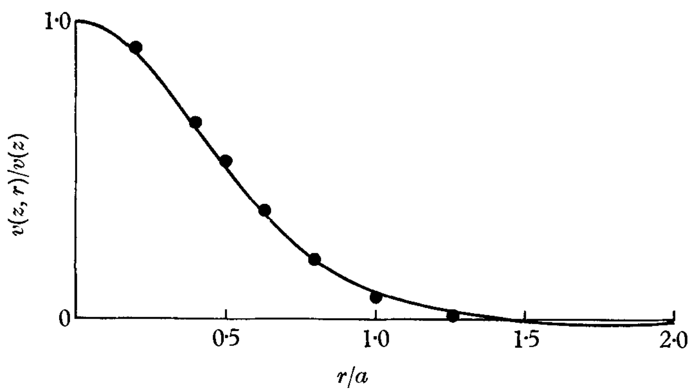

FIGURE 4. The vertical velocity profile relative to the fluid at rest, for potential flow around a moving sphere, at the level of its top or bottom (full line). This is compared with points calculated for a Gaussian profile having the same maximum and a length scale equal to the 'mean radius' R of a spherical vortex. ———, Potential flow; ●, Gaussian profile.

F and P will be increased by the flux of plume fluid into the rear of the cap. In order to evaluate fluxes relative to the moving cap, we must first decide how to specify the velocity profile across it.

Since the cap merges gradually with the plume behind it, the exact form to be taken for the cap is somewhat arbitrary. We could assume that it resembles a thermal with its slightly flattened shape, but in the absence of a definite reason for doing otherwise it is simplest for our purpose to adopt the model of the spherical vortex proposed by Levine (1959), for which quantitative results are known. Outside the vorticity containing region, the motion is like that past a solid sphere of the same radius as the vortex (Lamb 1932), so using standard results for potential flow about a sphere one can obtain the vertical velocity profile along a horizontal line through the top (or bottom) of the spherical vortex. This is drawn in figure 4, where it is compared with several points calculated for the Gaussian profile having the same maximum and a length scale equal to the 'mean radius', R, of the vortex, i.e. it is compared with $v(z)\, e^{-r^2/R^2}$ where $v(z)$ is the velocity of the sphere and $R^2 = \frac{2}{5}a^2$, a being the radius of the sphere. The agreement is surprisingly good except for the absence in the Gaussian profile of a downward moving region at large distances.

This result is the basis for the assumption which will now be made: that the junction between the cap and the plume occurs at the bottom of the spherical vortex and the velocity profile across the cap at this level remains Gaussian with a length scale equal to R and a velocity scale equal to the velocity of advance of the cap, even when the vortex is growing and mixing with its surroundings. The velocity scale used will be that appropriate to the base of the cap, say v, at height z_b. For an expanding spherical cap, this is related to the velocity of the

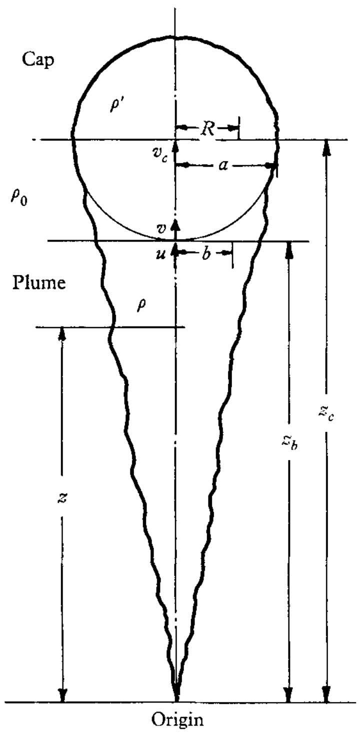

FIGURE 5. Diagram of the front of a starting plume, showing the relation between the various quantities referred to in §4.

centre v_c by $v = v_c(1-\alpha')$, where $\alpha' = a/z_c$ is defined as the 'half angle of spread' of the cap. The relation between the various heights and lengths used in this section is shown diagrammatically in figure 5. It seems fairly clear that R and the length scale for the plume b will not be very different, although these will be kept distinct in the development of the theory.

With this assumption the flux of buoyancy over a surface moving with the base of the cap may be calculated by integrating over Gaussian profiles. The time rate of increase of buoyancy in the cap is given by

$$dF/dt = \tfrac{1}{2}\pi Q\{1-(\beta v/u)\}, \tag{8}$$

where F and Q have been defined, and

$$\beta = 2/\{1+(b/R)^2\}. \tag{9}$$

The maximum velocities $u(z)$ and $v(z)$ are evaluated at the same height $z = z_b$ in the plume and the cap.

The momentum equation corresponding to (7) must now contain terms representing the increase of momentum due to buoyancy and also the flux of momentum from the plume. For Gaussian profiles the flux term can be evaluated directly and we find

$$dP/dt = \rho_0 F + \rho_0 \tfrac{1}{2}\pi b^2 u(u - \beta v). \tag{10}$$

The final equation necessary to determine P, R and z_b as functions of time may be written down if the relation between u and v is specified. It has already been implied that v must be constant fraction of u at the same height, and that β must be constant. This is in fact the only way the shape and size of the cap relative to the plume just below it can remain similar at all heights, and such a similarity assumption could have been taken as our starting-point. It has been more convenient to consider the two parts of the flow separately first, but we must now check that all the equations obtained above are consistent with this similarity condition.

Thus put

$$v(z) = c_1 u(z), \tag{11}$$

where $u(z)$ is given by (4) and c_1 is a constant (which later will be determined by experiment). Equation (8) may be integrated to give

$$F = \tfrac{1}{2}\pi Q(1 - \beta c_1)\, t. \tag{12}$$

In this, as in the earlier equations (8) and (10) describing the motion of the cap, t is the time which the base of the cap has taken to travel from the virtual origin to the height z_b. Integrating the equation for the velocity $v(z)$ obtained from (4) and (11) we obtain

$$t = \tfrac{3}{4}\tfrac{6}{5}\alpha c_1^{-1}(\tfrac{9}{5}\alpha Q)^{-\frac{1}{3}} z_b{}^{\frac{4}{3}}. \tag{13}$$

The relation (13) may now be used with (4) to evaluate the second term of (10) and to put it into the form

$$\begin{aligned} dP/dt &= \rho_0 F + \rho_0 \pi Q c_1(1 - \beta c_1)\, t \\ &= \rho_0 \pi Q(c_1 + \tfrac{1}{2})(1 - \beta c_1)\, t, \end{aligned} \tag{14}$$

using (12). Note that if $c_1 = \frac{1}{2}$, (14) implies that the two terms on the right-hand side contribute equally to the increase of momentum of the cap.

This last equation can now be integrated to give

$$P/\rho_0 = \tfrac{1}{2}\pi Q\,(c_1 + \tfrac{1}{2})(1 - \beta c_1)\, t^2. \tag{14a}$$

On substituting for t in terms of z_b using (13), and comparing with (5a) one obtains finally

$$R = \tfrac{6}{5}\alpha D z_b = Db, \tag{15}$$

where D is a geometrical constant given by

$$D^3 = \tfrac{5}{32}[(c_1 + \tfrac{1}{2})(1 - \beta c_1)\, c(1 - \alpha')]/\alpha c_1^3. \tag{16}$$

The factor $(1 - \alpha')$ arises because (5a) contains the velocity of the centre of the spherical cap. Thus a solution has been obtained which is consistent both with the plume and the vortex ring equations, and which relates the constants describing the behaviour of both parts of the flow. Note that since D and β

are both functions of R/b, α is a known constant for the plume (found by Morton *et al.* 1956 to be about 0·09) and α' may be put in terms of R/b and α, equation (16) shows that the behaviour of the cap may be specified by *two* extra constants, and the third obtained from these. Experimentally the easiest to measure are c_1, the ratio of the velocity of the base of the cap to the velocity in the plume at the same level, and R/b (or equivalently and more conveniently, the angle of spread α' of the visible edge of the cap), and these measurements are reported in § 5.

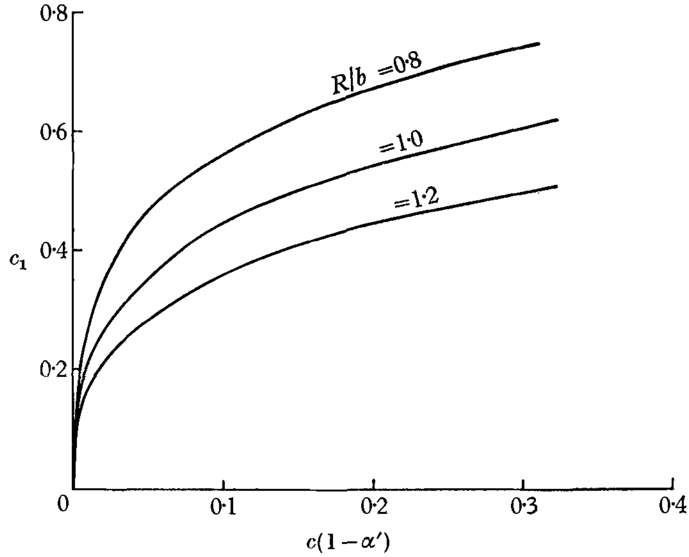

FIGURE 6. The relation between the constants $c(1-\alpha')$ and c_1 at the front of a starting plume, for several values of R/b. R is the 'mean radius' of the cap and b is the length scale of the plume at the level of the base of the cap. Equation (16) has been used with $\alpha = 0{\cdot}09$.

The above solution has already put certain limits on the values of the constants. It has been stated that c is likely to be not too different from its value in a thermal ($\approx 0{\cdot}15$) and that R and b should be comparable, so that the range of likely values of c_1 is restricted. This is shown in figure 6, where the relation between c_1 and $c(1-\alpha')$ is plotted for several values of R/b. The visible radius a will be of course greater than R; the factor is about $(5/2)^{\frac{1}{2}} = 1{\cdot}6$ for a spherical vortex and for Woodward's thermal of constant buoyancy.

It should also be remarked that, although the change of vorticity due to the action of buoyancy has not been considered specifically, the equations obtained here can be shown to be dimensionally consistent with the vorticity relations. The detailed discussion from this point of view would only introduce further geometrical constants which could be related to those used above.

5. Experimental results

Laboratory experiments have been carried out in order to measure directly the velocity of the cap compared with that of the plume behind, and the angle of spread of the cap. For convenience, the actual quantities obtained have been the velocity of the extreme front of the cap, and the angle of spread of the visible edge; these can be compared with the theory by reducing them to the values appropriate to the base of the cap and the 'mean radius'. We note that the edge of the cap is sharp, as it is in a thermal, and there are no difficulties in defining mean positions (as there is in a plume, because it is waving about).

The plume fluid consisted of salt solution, which was led from a roof tank to a downward pointing orifice held just below the surface of a large tank of fresh water. The plume was started suddenly by opening a tap near the orifice (so that heavy fluid immediately flowed out rather than being preceded by a jet of fresh water). This front was made visible with dye while the plume fluid behind was colourless (the photographs shown in figures 2 and 3, plates 2 and 3, in which the supply of dye was continued, were taken for illustration only). The tap setting was left unchanged, and when the front was well clear of the orifice, another pulse of colour was injected into the plume so that the motion of the steady flow could be followed. The whole operation was recorded on film.

The half angle of spread α' of the edge of the cap was measured from the film; note that this is defined as the radius at any height divided by that height (rather than by the distance to the front, as it has been usually for thermals). The mean value of 18 runs with a variety of flow rates was $0{\cdot}18 \pm 0{\cdot}03$; if the pattern of circulation in the cap is similar to that in a thermal or spherical vortex (if in particular the ratio of 'mean radius' R to the visible radius a is assumed to be the same) this result implies that $R/z_c = 0{\cdot}11$. This should be compared with the 'half angle of spread' for a plume, $b/z_b = \frac{6}{5}\alpha = 0{\cdot}11$; comparing R and b at the same height we see that $R/b \approx 1{\cdot}2$. It will later be shown (§ 6) that the constant c for the cap turns out to be close to that for a thermal released from rest, so there is no reason to doubt that the above assumption about the pattern of circulation is a fairly good one.

The half angle of spread of the visible edge may also be compared with that of a thermal, which is about $0{\cdot}3$ if the heights to the centre rather than the top are used. Thus the advancing front of a starting plume spreads at a considerably smaller angle than an unmaintained thermal released from rest.

It was intended that the velocity of the steady plume, as well as that of the front, should be measured from the films, but it was difficult to follow on 16 mm. film the successive positions of a given patch of dye as it diffused through the centre of the turbulent plume. Instead, the measurement was made by following the motion visually and timing with a stop watch between fixed marks. This was done first for the front, and then for a filament of dye in the steady flow behind, which latter gave a measure of a mean particle velocity up the centre of the plume. For this purpose it does not matter that the velocity is decreasing over the path chosen, since we are interested primarily in obtaining the *ratio* between the two velocities, and we have found that the functional dependence on distance is similar.

By varying the density and rate of output of salt, the velocity was changed by a factor of 3, implying a rate of output of density difference changing by a factor of 30. The mean ratio of front velocity to particle velocity for 39 runs was $0{\cdot}61 \pm 0{\cdot}05$. There is some uncertainty in the calculation of the velocity of the 'centre' (or base) of the cap from this, since the depth to be assigned to a region which merges with the plume below is not unambiguous. The main numerical uncertainties in the application of our theoretical model will arise from this cause. If we assume, however, that the region is spherical, then the velocity of the top of the sphere is $(1+\alpha')$ times the velocity of the centre, and the velocity in the plume changes by a factor of $(1+\alpha')^{-\frac{1}{3}}$ between these levels. Allowing for the velocity changes in both the plume and the cap, the ratio of the velocity of the centre to that on the axis of the established plume at this level is

$$0{\cdot}61 \times (1+\alpha')^{-\frac{4}{3}} = 0{\cdot}61 \times (1{\cdot}18)^{-\frac{4}{3}} = 0{\cdot}49.$$

If the cap is flattened the value is slightly greater than this, and if elongated, slightly less. The corresponding ratio between the velocity of the base of the cap and the central plume velocity there is $c_1 = 0{\cdot}38$.

The films of the advance of the front were also used to investigate directly the power-law dependence of the position of the front on the time. For a plume, $z \propto t^{\frac{3}{4}}$ from (4), and for a thermal released from rest, $z \propto t^{\frac{1}{2}}$. It is also relevant to recall that $u \propto z^{-1}$ and therefore $z \propto t^{\frac{1}{2}}$ for a jet (of zero buoyancy) in a uniform fluid. If the front of a starting plume behaves like the plume behind it, this should be distinguishable by direct measurements.

The origin of z was found by extrapolating the measured radii back to zero, and the origin of t adjusted until the best straight line was obtained on a log-log plot. The slope of this line was taken as the required exponent. The mean slope for the 16 experiments analysed in this way was $0{\cdot}72 \pm 0{\cdot}06$. This of course is not a very precise method of analysis, but it seems adequate to show that the velocity of the front of a starting plume behaves like that of the plume behind it, rather than that of a thermal.

In this connexion it is of interest to refer to some observations in the atmosphere which can be regarded as a large-scale test of the above ideas. At the beginning of certain volcanic eruptions a plume of smoke is ejected, and the cauliflower-like top of this has been photographed at successive times. In a recent review article, Sakuma & Nagata (1957) analyse the results of such measurements and find the power-law dependence is about $z \propto t^{0{\cdot}6}$. This is not consistent either with a neutral jet or with a thermal of constant buoyancy, but can be explained if one assumes that the behaviour of the front is somewhere between that of a jet and a plume. That is, the velocities may be initially so great that buoyancy can be neglected but later this becomes increasingly important. No detailed results have been found from which the other quantities available in the laboratory experiments could be obtained for the atmospheric case.

6. The fraction of entrained fluid entering from below

We have obtained the result that the angle of spread is less for a buoyant element maintained from below than it is for a thermal released from rest. This implies that the proportional rate of mixing per unit height E is also smaller, since by definition

$$E = V^{-1} dV/dz = 3\alpha'/a,$$

where V is the volume of the element, supposed spherical, and a is its radius. The difference is even greater than it appears when we think only of the rate of spread, since for a thermal all the fluid entrained comes directly from the environment, whereas at the front of the starting plume some environmental fluid is entrained directly but a considerable fraction comes from the plume below. The experimental results can now be used to calculate the fraction coming from the two sources.

The calculation will be carried out assuming that the cap is spherical; the qualitative changes to be made for other shapes will be mentioned. First of all it should be noted that the experimental values of $R/b = 1{\cdot}2$ and $c_1 = 0{\cdot}38$ correspond (equation (16)) to $c(1-\alpha') = 0{\cdot}11$ or $c = 0{\cdot}14$. This is very close indeed to the thermal value, much closer in fact that we could reasonably have expected in view of the uncertainties in the model and the sensitivity of c to R/b and c_1.

The volume V of the cap may be written as $V = qR^3$, where R is the 'mean radius' and q is a geometrical constant which is 16·6 for a spherical vortex and about 13 for a flattened thermal. The rate of change of V may therefore be expressed in the form

$$\frac{dV}{dt} = 3qR^2\frac{dR}{dz_b}\frac{dz_b}{dt} = 3q\,(\tfrac{6}{5}\alpha)^3 D^3 z_b^2 c_1 u, \qquad (17)$$

using (11) and (15). The rate of volume flow dV_1/dt from below may be evaluated by integrating across the Gaussian profiles (as was done to obtain equations (8) and (10)) and it follows that

$$dV_1/dt = \pi b^2(1-\beta c_1)\,u = \pi(\tfrac{6}{5}\alpha)^2 z_b^2(1-\beta c_1)\,u. \qquad (18)$$

Comparing (17) and (18), and substituting for D from (16) we have

$$\frac{dV_1}{dV} = \frac{16}{9}\frac{\pi}{q}\frac{c_1^2}{(c_1+\frac{1}{2})\,c\,(1-\alpha')}. \qquad (19)$$

Substituting the values appropriate to a spherical cap in (19), namely $q = 16{\cdot}6$, $c_1 = 0{\cdot}38$, $c = 0{\cdot}14$, $\alpha' = 0{\cdot}18$, we obtain $dV_1/dV = 0{\cdot}49$; that is, about half the fluid entering the spherical cap at any instant comes from below.

It is clear that the density difference between the cap and its surroundings will fall off less rapidly with distance than it does in a thermal, and in fact this property too will follow the plume equations (14). The mean density difference will be a constant fraction of that in the plume immediately below the cap; using an argument similar to that above it can in fact be shown that for a spherical cap the value is about $0{\cdot}5\,\Delta(z)$.

7. The two-dimensional case

A two-dimensional starting plume can be investigated in the same way, though the details will not be discussed here. Again it is possible to obtain a similarity solution, with the front advancing at a constant fraction of the plume velocity (which is independent of distance in this case).

A modification of this theory might also be applied to the 'nose' at the front of a layer of heavy fluid flowing down a sloping bed at high Reynolds numbers. It is to be expected that the ratio of the front to layer velocities, and also the shape of the nose, will depend on the slope and to a lesser extent on wall friction, in a way which would have to be determined experimentally in each case.

8. Application to the atmosphere

It has been shown how a theoretical description of the front of a 'starting plume' may be obtained, which is consistent both with the plume equations and with those describing a thermal-like element of increasing buoyancy. The laboratory experiments have given numerical values of two constants appearing in the theory, and we shall now summarize the results obtained and point out their implications in the context of cloud physics in which we wish to apply them.

(1) The centre of the cap moves at about half the maximum velocity in the plume behind it, and the velocity of the cap follows the power law appropriate to the plume rather than to a thermal of constant buoyancy.

(2) The half angle of spread of the visible edge of the cap is $\alpha' = 0{\cdot}18$, which is much less than that for a thermal. This result implies that the proportional rate of mixing into the cap per unit height, which is proportional to α', is just over half that for a thermal of constant buoyancy.

(3) About half the fluid added to the cap comes from the plume, rather than directly from the environment. This implies a further decrease in the effective rate of dilution of fluid in the cap, and the density difference will in fact have the same dependence on distance as it does in a plume.

We have suggested that these results might be used to investigate the effects of mixing from above and below on the properties of small clouds, and they are certainly directly applicable to the dry convection plume below cloud base. For such a plume we could determine the distribution with height of moisture as well as temperature, in the way that Morton (1957) has done for *steady* plumes in an atmosphere with a known moisture distribution. Clearly the equations of conservation of water vapour can be used, with the numerical values provided by the experiments reported here, to carry out similar calculations for the front of a starting plume and to work out where condensation will occur.

No detailed calculations of this kind will be attempted here. It should be remarked that results based on point sources at the ground are of limited value in predicting the height of cloudbase, since they imply far too high a temperature at any level. In practice, convection will begin when air near the ground is heated at a slow rate over a large area, and the condensation level will be raised, because of mixing, only a little above the height predicted by supposing that air is lifted from the ground without mixing. In this respect the starting plume will be little different from the steady plume.

The starting plume model does, however, allow us to make an estimate of a quantity which has not previously been considered, namely, the time for which a steady source of heat must operate to produce a cloud at a known level. We shall conclude with a simple numerical example to illustrate the method.

Suppose that the condensation level is at $1\frac{1}{2}$ km above the ground and that the front of the plume responsible for the formation of a cloud is 1 km diameter at that level. Extrapolating downwards we can say that the starting plume has arisen from an area 460 m in diameter, or from a virtual source 1·3 km below the ground. If the heating over this area of ground is at the rate of 0·2 cal/cm^2 min, say, it can be shown that Q (defined by equation (3)) is about 5×10^{10} c.g.s. units. Applying equation (13), with appropriate changes for the finite source, we find that the centre of the 'cap' of the starting plume will reach $1\frac{1}{2}$ km in 690 sec $= 11\frac{1}{2}$ min, and its mean upward velocity is therefore about 2 m/sec.

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FIGURE 1. Photograph of a cloud which has formed on top of a smoke plume produced by a grass fire.

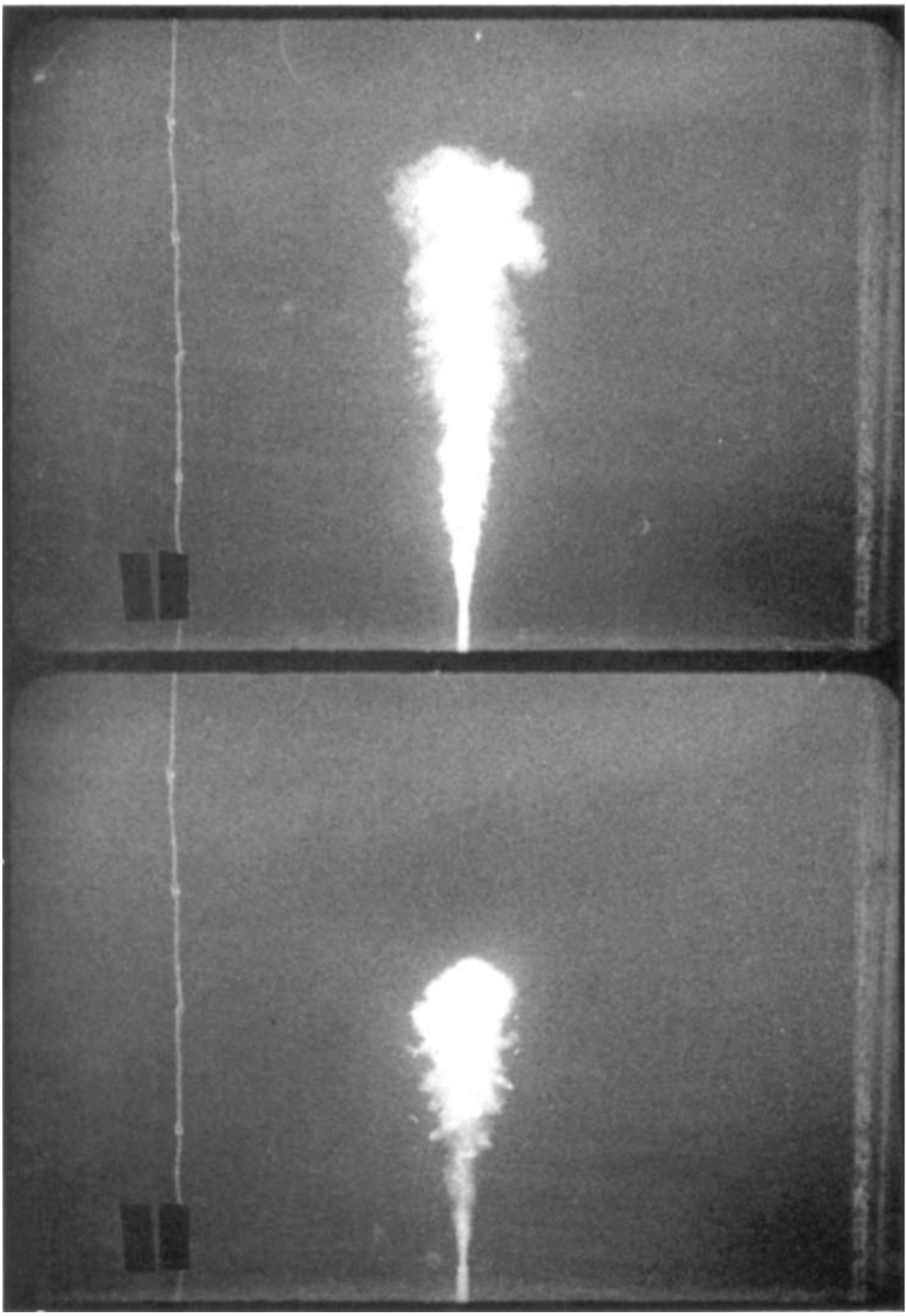

FIGURE 2. Successive pictures taken 1 sec apart of a starting plume in a laboratory tank. The plume fluid consisted of dyed salt solution moving downwards through fresh water, but these photographs and figure 3 are printed inverted for easier comparison with figure 1.

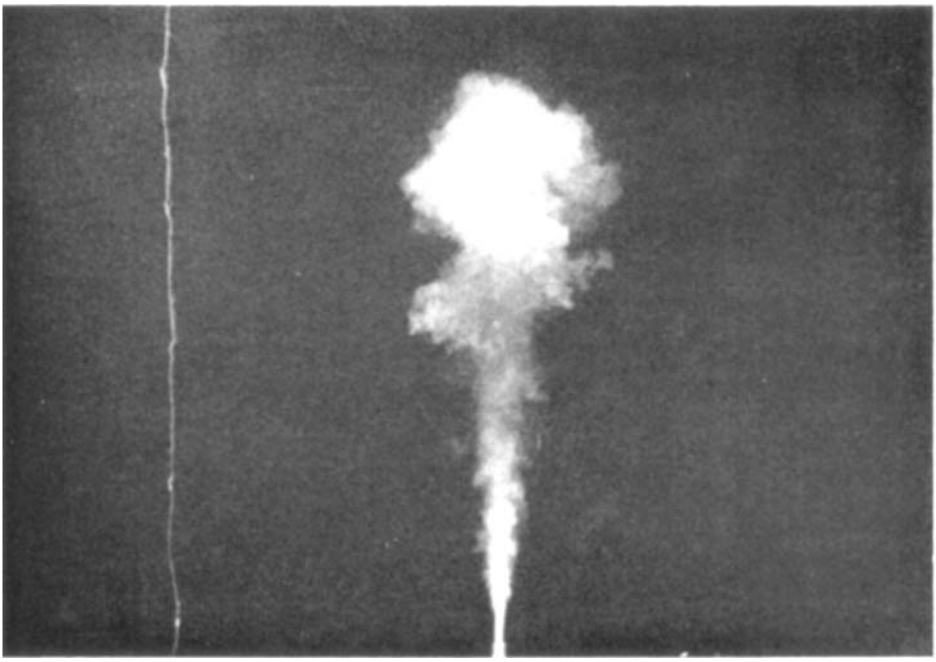

FIGURE 3. Starting plume in which the supply of dye has been momentarily interrupted to show the front, which looks very like a thermal, and the steady plume behind.